INDIAN SCHOOL MUSCAT
FINAL EXAMINATION 2020-21
MATHEMATICS

CLASS: XII   Sub. Code: 041   Time Allotted: 3 Hrs.

24.01.2021   Max. Marks: 80

General Instructions:

1. This question paper contains two parts A and B. Each part is compulsory. Part A carries 24 marks and Part B carries 56 marks

2. Part-A has Objective Type Questions and Part -B has Descriptive Type Questions

3. Both Part A and Part B have choices.

Part – A:

1. It consists of two sections- I and II.

2. Section I comprises of 16 very short answer type questions.

3. Section II contains 2 case studies. Each case study comprises of 5 case-based MCQs. An examinee is to attempt any 4 out of 5 MCQs.

Part – B:

1. It consists of three sections- III, IV and V.

2. Section III comprises of 10 questions of 2 marks each.

3. Section IV comprises of 7 questions of 3 marks each.

4. Section V comprises of 3 questions of 5 marks each.

5. Internal choice is provided in 3 questions of Section –III, 2 questions of Section-IV and 3 questions of Section-V. You have to attempt only one of the alternatives in all such questions.

## PART A

SECTION I

1. The relation R be defined on the set $A = \{1, 2, 3, 4, 5\}$ by $R = \{(a, b) : |a^2 + b^2| < 8 \}$, then write the relation R in roster form.   1

2. How many equivalence relations are possible in a set A where $n(A) = 2$ 1

3. If $n(A) = 6$ and $n(B) = 7$, then find the number of bijective functions from A to B.   1

4. For what value of k, the matrix $\begin{bmatrix} -4 & 2 \\ k+1 & k-5 \end{bmatrix}$ does not have an inverse?   1

5. If A and B are matrices of order 2 and $|A| = 6$, $|B| = 3$, *then find* $|4AB|$ 1

6. Write the number of all possible matrices of order 2 x 2 with each entry 0, 1, 2 or 3 .   1

7. $\int_{-2}^{1} \frac{|x|}{x} dx$ **OR**   If $\int \frac{1}{\sqrt{9-16x^2}} dx = \frac{1}{4}\sin^{-1}(ax) + c$, find the value of a   1

8. Find the area of the region bounded by the curve x = 2y + 3 , Y-axis and the lines y = 1   1

OR

Find the area bounded by the curve $y = x|x|$, x- axis and the lines x = -2 and x = 2

9. Find the sum of order and degree of the differential equation $\frac{d}{dx}\left\{ \left(\frac{dy}{dx}\right)^3 \right\} = 0$ 1

OR

Find the integrating factor of differential equation $\left(\frac{e^{-2\sqrt{x}}}{\sqrt{x}} - \frac{y}{\sqrt{x}} \right)\frac{dx}{dy} = 1$

10. Find the value of β for which the vectors $3\hat{\imath} - 6\hat{\jmath} + \hat{k}$ and $4\hat{\imath} - 8\hat{\jmath} + \beta\hat{k}$ are parallel.   1

11. What is the distance of the point (1, 4, 7) from the X – axis?   1

12. The two vectors $\hat{\jmath} + \hat{k}$ and $3\hat{\imath} - \hat{\jmath} + 4\hat{k}$ represent the two sides AB and AC, respectively of a ΔABC. Find the length of the median through C.   1

13. Find the reflection of the point (2, 3, 5) in the YZ plane.   1

14. Find the projection of $\vec{a}$ on $\vec{b}$ if $\vec{a} . \vec{b} = 8$ and $\vec{b} = 2\hat{\imath} + 6\hat{\jmath} + 3\hat{k}$.   1

15. If A and B are two events such that P(A) + P(B) – P(A and B) = P(A), then find the value of P(A/B).   1

16. The probability distribution of the discrete variable X is given below, find the value of k.   1

X	1	2	3	4	5
P(X)	$\frac{1}{k}$	$\frac{3}{k}$	$\frac{4}{k}$	$\frac{5}{k}$	$\frac{3}{k}$

SECTION II

17. Mr Kumar, who is an architect, designs a building for a small company. The design of window on the ground floor is proposed to be different than other floors. The window is in the shape of a rectangle which is surmounted by a semi-circular opening. This window is having a perimeter of 10 m as shown below :   4
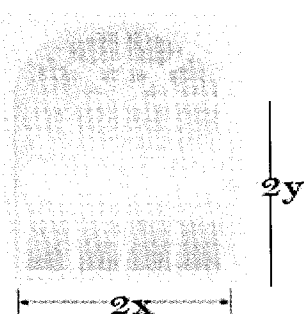
Based on the above information answer the following :

(i) If 2x and 2y represents the length and breadth of the rectangular portion of the windows, then the relation between the variables is

(a) $4y - 2x = 10 - \pi$

(b) $4y = 10 - (2 - \pi)x$

(c) $4y = 10 - (2 + \pi)x$

(d) $4y - 2x = 10 + \pi$

(ii) The combined area (A) of the rectangular region and semi-circular region of the window expressed as a function of x is

(a) $A = 10x + (2 + \frac{1}{2}\pi)x^2$

(b) $A = 10x - (2 + \frac{1}{2}\pi)x^2$

(c) $A = 10x - (2 - \frac{1}{2}\pi)x^2$

(d) $A = 4xy + + \frac{1}{2}\pi x^2$

(iii) The maximum value of area A, of the whole window is

(a) $A = \frac{50}{\pi - 4} m^2$

(b) $A = \frac{50}{\pi + 4} cm^2$

(c) $A = \frac{100}{\pi + 4} m^2$

(d) $A = \frac{50}{4 - \pi} m^2$

(iv) The owner of this small company is interested in maximizing the area of the whole window so that maximum light input is possible. For this to happen, the length of rectangular portion of the window should be

(a) $\frac{20}{\pi + 4} m$

(b) $\frac{10}{\pi + 4} m$

(c) $\frac{4}{\pi + 10} m$

(d) $\frac{100}{\pi + 4} m$

(v) In order to get the maximum light input through the whole window, the area (in sq. m) of only semi-circular opening of the window is

(a) $\frac{100\pi}{(4+\pi)^2}$

(b) $\frac{50\pi}{4+\pi}$

(c) $\frac{50\pi}{(4+\pi)^2}$

(d) same as the area of rectangular portion of the window

18. An insurance company insure three type of vehicles i.e., type A, B and C. It insured 12000 vehicles of type A, 16000 vehicles of type B and 20,000 vehicles of type C. Survey report says that the chances of their accident are 0.01, 0.03 and 0.04 respectively.

Based on the informations given above, write the answer of following:

4

(i)   The probability of insured vehicle of type C is

     (a) $\frac{5}{12}$ (b) $\frac{4}{12}$ (c) $\frac{7}{12}$ (d) $\frac{3}{12}$

(ii)   Let E be the event that insured vehicle meets with an accident then P(E/A) is

     (a) 0.09 (b) 0.01 (c) 0.07 (d) 0.06

(iii)   Let E be the event that insured vehicle meets with an accident then P (E) is

     (a) $\frac{38}{1200}$ (b) $\frac{32}{1200}$ (c) $\frac{24}{1200}$ (d) $\frac{35}{1200}$

(iv)   The probability of an accident that one of the insured vehicle meets with an accident and it is a type C vehicle

     (a) $\frac{2}{7}$ (b) $\frac{3}{7}$ (c) $\frac{5}{7}$ (d) $\frac{4}{7}$

(v)   One of the insured vehicles meets with an accident and it is not of type A and C.

     (a) $\frac{12}{35}$ (b) $\frac{20}{35}$ (c) $\frac{1}{35}$ (d) $\frac{17}{35}$

PART B
SECTION III

19. Find the value of $tan^2(sec^{-1} 2) + cot^2(cosec^{-1} 3)$.   2

20. $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$ Show that $A^2 - 5A + 7I = 0$. Hence find A^{-1}.   2

OR

Express the following matrices as the sum of a symmetric matrix and a skew symmetric matrix:

$$C = \begin{bmatrix} 3 & 5 \\ 1 & -1 \end{bmatrix}$$

21.   2

If f(x) = $\begin{cases} x + a\sqrt{2}\ sinx , 0 \le x \le \frac{\pi}{4} \\ 2xcotx + b, \frac{\pi}{4} \le x < \frac{\pi}{2} \\ acos2x - bsinx , \frac{\pi}{2} \le x < \pi \end{cases}$ is continuous on $[0, \pi]$, find the values of a and b

OR

If the following function is differentiable at x = 2 , then find the values of a and b

$$f(x) = \begin{cases} x^2, if\ x \le 3 \\ ax + b, \ if\ x > 3 \end{cases}$$

22. Find the point on the parabola $f(x) = (x - 3)^2$, where the tangent is parallel to the chord joining the points (0, 3) and (4, 1).   2

23. Evaluate $\int secx$ dx   2

     OR

Evaluate $\int \frac{1}{1+tanx} dx$

24. Find the area bounded by the lines $y = x+2$, $x = 2$ and Y-axis, using the method of integration. 2

25. Find the general solution of the following differential equation: $\log\left(\frac{dy}{dx}\right) = ax + by$. 2

26. Find the area of the parallelogram whose diagonals are represented by the vectors 2
$3\hat{\imath} + \hat{\jmath} - 2\hat{k}$ and $\hat{\imath} - 3\hat{\jmath} + 4\hat{k}$.

27. Find the shortest distance between two Lines 2
$\vec{r} = \hat{\imath} + \hat{\jmath} + \lambda(2\hat{\imath} - \hat{\jmath} + \hat{k})$ and $\vec{r} = 2\hat{\imath} + \hat{\jmath} - \hat{k} + \mu(3\hat{\imath} - 5\hat{\jmath} + 2\hat{k})$

28. A problem in mathematics is given to 3 students whose chance of solving it are 1/3, 1/4 and 1/5. 2
What is probability that the problem is solved?

OR

In a school, there are 1000 students, out of which 430 are girls. It is known that out of 430, 10% of the girls study in class XII. What is probability that a student chosen randomly studies in class XII given that the chosen student is a girl.

SECTION IV

29. Show that the relation R in the set A={1,2,3,4,5,6,7,8,9,10} given by 3
R={(a, b): $|a - b|$ is an even number} is an equivalence relation. Write equivalence class [3]

30. Find $\frac{dy}{dx}$, if $(\sin x)^y = (\sin y)^x$ 3

OR

For a positive constant a , find $\frac{dy}{dx}$ if $x = \left(t + \frac{1}{t}\right)^a$ and $y = a^{\left(t + \frac{1}{t}\right)}$

31. If $x^{16}y^9 = (x + y)^{25}$, then prove that $\frac{dy}{dx} = \frac{y}{x}$. 3

32. Show that the function f is given by $f(x) = \tan^{-1}(\sin x + \cos x)$, $x > 0$, is always a strictly 3
increasing function in $(0, \frac{\pi}{4})$.

33. Evaluate : $\int_0^\pi \frac{x}{a^2\cos^2 x + b^2\sin^2 x} dx$ 3

34. Find the area of the region bounded by the curve $y = \sqrt{25 - x^2}$ and x-axis using integrals. 3

OR

Find the area of the region bounded by the curve $y + 4 = x^2$ and x-axis using integrals.

35. *Solve the differential equation* $\cos^3 x \frac{dy}{dx} + y\cos x = \sin x$ $(0 \leq x < \frac{\pi}{2})$ when $y = 1$, $x = \frac{\pi}{4}$ 3

SECTION V

36. 10 students were selected from a school on the basis of value for giving awards and were divided 5
into three groups. The first group comprises hard workers, the second group has honest and law abiding students and the third group contain vigilant and obedient students. Double the number of students of the first group and added to the number in the second group gives 13, while the combined strength of the first and second group is four times that of the third group. Using matrix method, find the number of students in each group.

OR

If $A = \begin{bmatrix} 1 & -1 \\ 2 & -1 \end{bmatrix}$, $B = \begin{bmatrix} a & -1 \\ b & -1 \end{bmatrix}$ and $(A + B)^2 = A^2 + B^2$, then find a and b.

37   Find the equation of the plane passing through the points (1, 2, -1) and (2, 0, 2) and parallel to the   5
line $\vec{r} = (2\hat{i} + \hat{j} + 2\hat{k}) + \lambda(\hat{i} + 2\hat{j} + 2\hat{k})$

OR

Find the foot of the perpendicular from P(1, 2, 3) on the line $\frac{x-6}{3} = \frac{y-7}{2} = \frac{z-7}{-2}$. Also obtain the

equation of the plane containing the line and the point (1, 2 ,3).

38.   Solve the following linear programming problems graphically: 5

Minimize and Maximize Z = 5x + 10y subject to x + 2y ≤ 120, x + y ≥ 60 , x – 2y ≥ 0, x ,y ≥ 0

OR

Minimize Z = 10(x – 7y + 190) subject to the constraints: x + y ≤ 8 , x ≤ 5 ,y ≤ 5 , x + y ≥ 4 ,
x ,y ≥ 0

End of the Question Paper